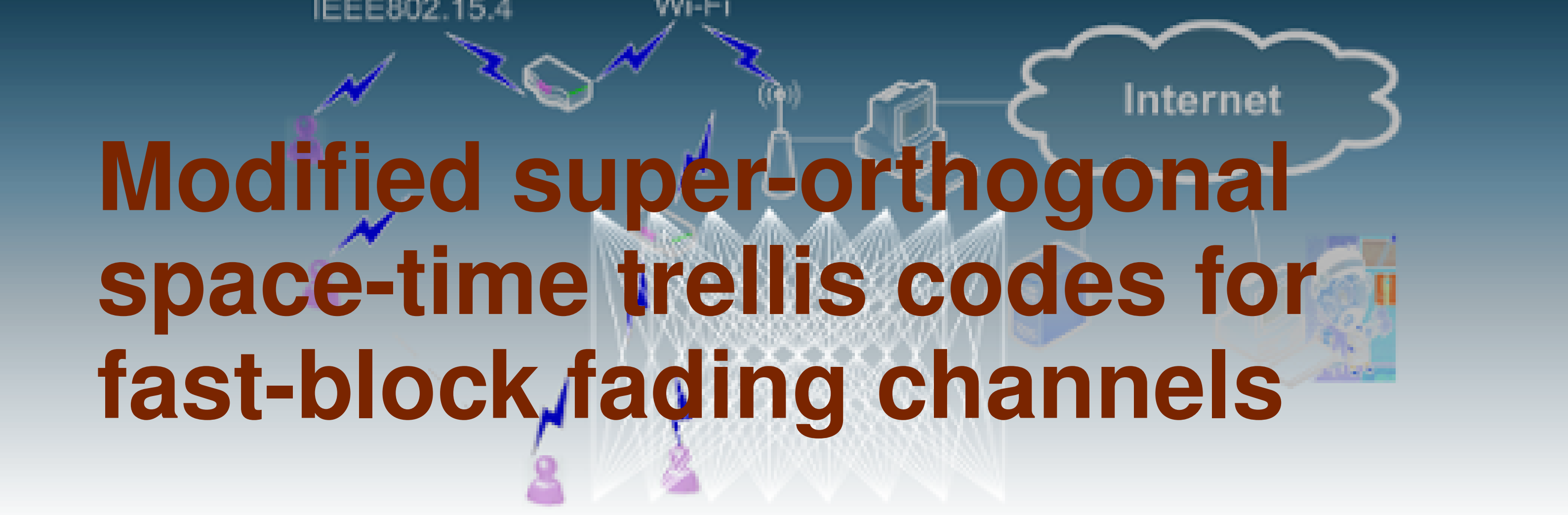

Modified super-orthogonal space-time trellis codes for fast-block fading channels

Jorge Flores-Troncoso, Leonel Soriano Equigua, Jaime Sánchez-García
Salvador Ibarra-Delgado, Remberto Sandoval-Aréchiga, José Luis Alvarez-Flores,
and José Manuel Cervantes Viramontes

Códigos en enrejado espacio-tiempo super-ortogonales modificados para canales con desvanecimiento rápido

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Palabras clave: Códigos espacio-tiempo; códigos en enrejado; desvanecimiento rápido.

Abstract:
In this paper, we present new super orthogonal space-time trellis codes (Mo-SOSTTCs) for channels modeled as fast block-fading, where the channel is constant for each codeword matrix and changes independently from one codeword to the next. We introduce an extra rotation parameter in a codeword matrix for expanded constellation which allows to minimize the parallel paths in the trellis without increasing the complexity in the decoder. Like original super-orthogonal space-time trellis codes (SOSTTCs), these codes combine set partitioning and a super set of orthogonal space-time block codes, but unlike SOSTTCs we use the product distance criterion instead of coding gain distance criterion to define a set partitioning. Moreover, we also use the symbol Hamming distance and rank criteria together. Simulations results show that the proposed Mo-SOSTTCs provide similar performance than that of the existing SOSTTCs in quasi-static channel case. Furthermore, Mo-SOSTTCs provide better performance in fast block-fading channel.

Keywords: Space-time codes, trellis codes, fast fading.

SPACE-TIME codes provide an effective method to increase system capacity for wireless communications. Tarokh, et al., first described the performance characteristics of space-time codes in terms of diversity gain and coding gain [1]. As the diversity gain determines the asymptotic slope of the Pair-wise error probability (PEP) in logarithm domain, it is the most important factor in the design of space-time codes. Therefore, the design of the earliest space-time trellis codes has been focused primarily on implementing full diversity order, but non-optimal coding gain [2]. The various improved codes have been developed to enhance the coding gain performance in subsequent works. The improvement may be implemented by finding more powerful codes that yield the minimum PEP in terms of Tarokh's criteria through exhaustive computer search [3], developing improved design criteria based on more accurate mathematical expression of PEP [4][5], and performing union bound and distance spectrum analysis [6][7]. Nowadays, with the development space-time codes as the state of art techniques, these methods may now only provide marginal improvement.

Space-time codes provide an effective method to increase system capacity for wireless communications

The improvement can be further increased by introducing the orthogonal design of space-time block codes into space-time trellis codes. In slow fading channels, the coding gain depends on the minimum determinant of a full rank codeword distance matrix [1]. Thus, the optimal determinant will be obtained if the codeword distance matrix is also a diagonal matrix with equal eigenvalues.

A systematic method to design the trellis codes for quasi-static fading channel case was shown in [8], called super-orthogonal space-time trellis codes (SOSTTCs). It consists on maximizing the coding gain combining space-time block codes with a trellis code to come up with a new structure that guarantees the full diversity with any given rate and number of states. SOSTTCs outperform the previously discussed Space-Time Trellis Codes (STTCs) in [1] by more than 2 dB. Moreover, the decoding complexity of SOSTTCs is smaller than that of STTCs.

SOSTTCs are known to be a very effective coding technique for improving coding gain performance in slow fading channels. However, these codes are not ideally suited for fast fading channels, because they may give rise to the diversity loss in these scenarios [9]. In SOSTTCs, the super-trellis transition is induced by M_t information symbols. This super-trellis operation ensures full rate codes, but inevitably results in parallel trellis transitions. In the fast fading channel case, diversity gain depends on Minimum Symbol Hamming Distance (MSHD) [1]. The symbol Hamming distance measures the number of different symbol positions between two codewords. Thus, the STTCs that exhibit parallel trellis transitions have a MSHD with a maximum value of one. Although the practical MSHD is 2 because of the use of Alamouti matrix as the encoded output in SOSTTC, compared to its counterpart with the same number of states but MSHD greater than 2 in the STTCs, SOSTTC would suffer diversity loss in fast fading channels because of their rotation structure.

Generally speaking, it is very difficult to prevent parallel trellis transition from happening, especially for the systems that have a large number of transmitting antenna and/or a high order constellation modulation.

The main contribution of this paper is a new class of SOSTTC which satisfies the design criteria in fast-block fading channels, as proposed by Tarokh *et al.* Through analysis and numerical simulations, we demonstrate that the proposed modified-SOSTTCs outperform the knows SOSTTCs with the canal conditions established. The rest of paper is organized as follows: first we provide the system and channel model. Then we study the performance analysis and show the design criteria for the proposed modified super-orthogonal space-time trellis codes. Afterwards, we show the set partitioning and code design using two transmitting antennas for BPSK and QPSK constellations. Next we explain important properties of proposed codes. Then, we present numerical simulation results. Finally, the conclusions are presented in last section.

MODIFIED-SOSTTC

System model

Since the diversity gain of space-time codes corresponds to the asymptotic slope of the PEP, any diversity loss can result into significant performance degradation.

Thus, diversity loss is an important issue to be taken into account in space-time codes. We have, therefore, developed the Modified Super Orthogonal Space-Time Trellis Codes (Mo-SOSTTCs) in this work, which are aimed at improving diversity gain in fast block-fading channels. The approach is also based on the orthogonal design of space-time block codes. But unlike SOSTTCs, the parallel transitions are avoided or minimized in our design. Theoretical analysis and simulation results indicate that in fast block-fading channels, Mo-SOSTTCs have an advantage over SOSTTCs regarding both diversity gain and coding gain.

Like the SOSTTC, at each time slot k, a parallel b bit information data stream is injected into the encoder, and these bits are then converted to M-PSK symbols, where $b = \log_2 \mathrm{M}$. We represent an output M-PSK symbol in ith transmitting antenna for $i = 1,2$ at time k by

$$x_k^i = e^{j\frac{2\pi}{M}m} e^{j\phi} \tag{1}$$

where $m = 0,1,\ldots,\mathrm{M}-1$ represents the index of the symbols in the space-time code, and ϕ is the rotation angle for expanded constellation which allows to obtain high coding gain. Then, the resultant $T \times M_t$ Alamouti orthogonal matrix [10] to be transmitted at time k in each branch transition takes the form

$$c_k = \begin{pmatrix} x_k^1 & x_k^2 \\ -(x_k^2)^* & (x_k^1)^* \end{pmatrix} \quad (2)$$

where $M_t = 2$ transmitting antennas. It is important to note that this is a special case of (5) in [11]. Note than an orthogonal space-time codeword consists of $2b$ information bits and is further divided into $T = 2$ time slots. The orthogonal matrix is transmitted row by row within time slot k, such that the ith column is transmitted from the ith antenna, and each row is transmitted in the corresponding time slot. This indicates that the number of transmitted symbols in one codeword matrix duration is 2. Therefore, the scheme has a rate of b bits/seg/Hz.

A systematic method to design the trellis codes for quasi-static fading channel case is called super-orthogonal space-time trellis codes (SOSTTCs)

Performance analysis

Performance of space-time codes has been analyzed by formulating upper bound on the PEP [1]. PEP refers to the probability that the decoded codeword at the receiver is different from the transmitted codeword. Let K be the length of frame, $\mathbf{c}$ the transmitted codeword and $\mathbf{e}$ the erroneously decoded codeword. Let c_k and e_k denote the encoded output matrix of size $T \times M_t$ at time k in codeword $\mathbf{c}$ and $\mathbf{e}$, respectively. We define codewords $\mathbf{c}$ and $\mathbf{e}$ such that each one is a matrix formed by concatenating c_k and e_k, respectively,

$$\mathbf{c} = \begin{pmatrix} c_1 & c_2 & \cdots & c_k & \cdots & c_K \end{pmatrix}^{\mathcal{T}} \quad (3)$$

$$\mathbf{e} = \begin{pmatrix} e_1 & e_2 & \cdots & e_k & \cdots & e_K \end{pmatrix}^{\mathcal{T}} \quad (4)$$

Let $\mathbf{D}_k = c_k - e_k$ be a branch difference matrix (since this matrix is associated with the branch output). A branch codeword distance matrix $\mathbf{A}_k$ between c_k and e_k is defined as the Hermitian square of the difference matrix of size $M_t \times M_t$

$$\mathbf{A}_k = \mathbf{D}_k^H \mathbf{D}_k \quad (5)$$

For the STTCs in [1], the pairwise error probability for fast fading case has been calculated considering that each trellis branch output is a symbol vector . However, when the branch output is a matrix as in Mo-SOSTTC, the channel coefficients are assumed to remain constant during the transmission of one $T \times M_t$ codeword c_k (fast block-fading channel), but they change in a random manner from one codeword to another. If $\mathbf{H}_k$ is the channel during the transmission of c_k, and r_k is the received signal block, then the maximum likelihood (ML) decoder output is given by

$$\widehat{\mathbf{c}} = \arg\min_{\mathbf{c}} \sum_{k=1}^{K} \|r_k - c_k \mathbf{H}_k\|_F^2, \quad (6)$$

where $\mathbf{c}$ is given by (3). Therefore, the upper bound of PEP in fast block-fading Rayleigh channels with M_r antennas at the receiver side can be written as [9]

$$P(\mathbf{c} \to \mathbf{e}) \leq \prod_{k=1}^{K} \prod_{i=1}^{r} \left(\frac{1}{1 + \frac{E_s}{4N_0} \lambda_k^i} \right)^{M_r} \quad (7)$$

where λ_k^i is the ith eigenvalue of $\mathbf{A}_k$ in descending order and r is the rank of $\mathbf{A}_k$. Let SNR be sufficiently large, hence the PEP expression in (7) can be approximated by

$$P(\mathbf{c} \to \mathbf{e}) \leq \left(\frac{E_s}{4N_0} \right)^{-\delta_H r M_r} \prod_{k \in \rho(\mathbf{c},\mathbf{e})} \left(\prod_{i=1}^{r} \lambda_k^i \right)^{-M_r} \quad (8)$$

where $\rho(\mathbf{c}, \mathbf{e})$ is the set of time instance at which the $\mathbf{A}_k$ is not a null matrix such that $c_k \neq e_k$ for $1 \leq k \leq K$; δ_H is the number of time instances in which the codeword pair $(\mathbf{c}, \mathbf{e})$ differ and it is known as the *Symbol Hamming Distance*.

Design criteria

We can see from (8) that, in the case of fast block-fading channels, the order of diversity is $\delta_H r M_r$, where $\min\{\delta_H\}$ is the minimum symbol Hamming distance (MSHD) between the different codewords (we have considered each c_k as a matrix symbol).

By observing the inequality (8), one can see that the product term of eigenvalues determines the coding gain. Recognizing that the product term of eigenvalues is equal to the determinant of matrix $\mathbf{A}_k$ (if $\mathbf{A}_k$ is a full-rank matrix), the design criteria to yield minimum PEP for Mo-SOSTTC are as follows:

Symbol Hamming distance criterion

The minimum symbol Hamming distance must be maximized.

Rank criterion

The branch codeword distance matrix $\mathbf{A}_k$, where $k \in \rho(\mathbf{c},\mathbf{e})$, must have a full rank of M_t. Using this criterion together with the above symbol Hamming distance criterion we get an optimal diversity gain.

Product distance criterion

In order to provide optimal coding gain, the minimum determinant of $\mathbf{A}_k$ must be maximized. As afore mentioned, the orthogonal matrix provides guaranteed full rank for $\mathbf{A}_k$. It can easily be proven that $\mathbf{A}_k$ can be expressed as

$$\mathbf{A}_k = \left(\left|c_k^1 - e_k^1\right|^2 + \cdots + \left|c_k^{M_t} - e_k^{M_t}\right|^2\right)\mathbf{I}_{M_t} \tag{9}$$

Expression (9) states that $\mathbf{A}_k$ is a full rank diagonal matrix, in which the diagonal elements are identical and equal to the Euclidean distance of the encoded outputs in STTCs. Thus, ideally the orthogonal matrix should be chosen to make the codes robust on both diversity and coding gain. In this case, from (9), it can be seen that the eigenvalues will be

$$\lambda_k^i = \left|c_k^1 - e_k^1\right|^2 + \left|c_k^2 - e_k^2\right|^2 + \cdots + \left|c_k^{M_t} - e_k^{M_t}\right|^2 \tag{10}$$

for $i = 1, 2, \ldots, M_t$.

Applying the property given in (10) to (8) yields

$$P(\mathbf{c} \to \mathbf{e}) \leq (\mathrm{dp2})^{-M_t M_r}\left(\frac{E_s}{4N_0}\right)^{-\delta_H M_t M_r} \tag{11}$$

where

$$\mathrm{dp2} = \prod_{k \in \rho(\mathbf{c},\mathbf{e})}\left(\left|c_k^1 - e_k^1\right|^2 + \cdots + \left|c_k^{M_t} - e_k^{M_t}\right|^2\right) \tag{12}$$

is the product distance of Mo-SOSTTC.

In [8] the authors defined the coding gain distance (CGD) between codewords c_k and e_k as the determinant of matrix $\mathbf{A}_k$. Unlike SOSTTCs, we use dp2 instead of CGD to define a set partitioning.

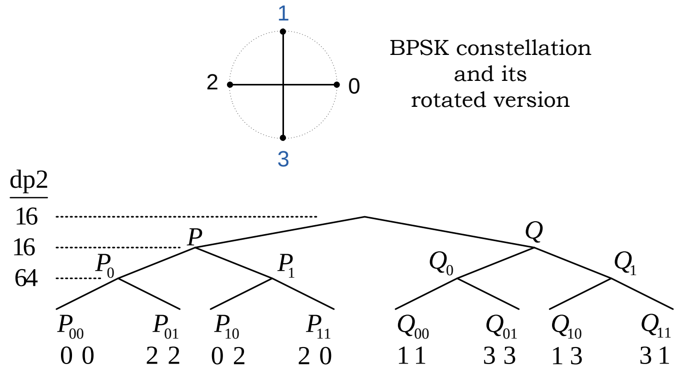

Figura 1. Set partition for BPSK over two transmitting antennas.

SET PARTITIONING AND DESIGN FOR Mo-SOSTTC

The code considered here was designed for two transmitting antennas for both BPSK and QPSK constellations in the fast block-fading scenario. For full-rate codes, we need $N_{cw} = 2M_t^{\mathrm{M}}$ codeword matrices at least. If $M_t = 2$, then $N_{cw} = 8$ for BPSK or $N_{cw} = 32$ for QPSK. In addition, in order to obtain a full-rate Mo-SOSTTC, we must increase the number of available orthogonal matrices trying to either prevent or minimize the parallel paths in the trellis. Hence, we will expand the size of the constellation alphabet of the transmitted signals, without affecting the full-diversity and coding gain for each pair of codes transmitted at each branch in the trellis, maintaining also the performance in the quasi-static channel case. Similarly to extended-SOSTTC [11], the Mo-SOSTTC utilizes one extra rotation parameter ϕ to expand the inner OSTBC, but it does not use the rotation angle θ as SOSTTC.

Let $\phi \in \{0, \frac{\pi}{\mathrm{M}}\}$ two rotation parameters in a codeword as given by (2), then we have $N_{cw} = 8$ OSTBCs for BPSK and $N_{cw} = 32$ OSTBCs for QPSK. When $\phi = 0$ we use P to represent the set of these 2^{M} OSTBCs, and we use Q to represent the other set of 2^{M} OSTBCs whit $\phi = \frac{\pi}{\mathrm{M}}$. Following the general method in [8], we partition the sets to design a trellis code maximizing the product distance dp2 between codeword pairs at each level, considering the worst case which arises when there are parallel transitions, but in a later step this will be minimized.

Figures 1 and 2 show the set partitioning for both BPSK ($r = 1$ bit/s/Hz) and QPSK ($r = 2$ bit/s/Hz) respectively with

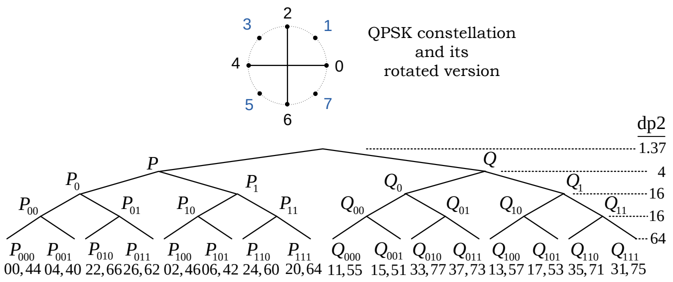

Figura 2. Set partition for QPSK using two transmitting antennas.

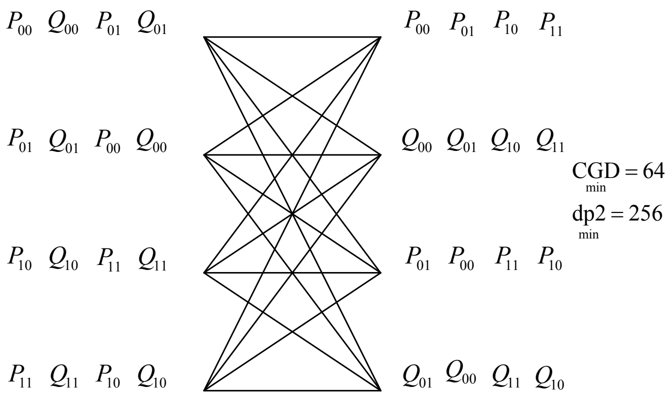

Figura 3. Four state code $M_t = 2$, $r = 1$ bit/s/Hz.

$M_t = 2$. The numbers at leaves represent the index of the symbols in the constellation MPSK to be transmitted in the orthogonal space-time code.

Now, we propose the design of full-rate Mo-SOSTTC with full diversity and high coding gain for two transmitting antennas. In order to achieve full diversity, we can see that unlike the SOSTTC, taking any codeword c_k of P with any codeword e_k of Q, the difference matrix $\mathbf{D}_k$ is a full-rank matrix. Therefore, the branch codeword distance matrix $\mathbf{A}_k$ is not a null matrix.

Suppose that the encoder is in a certain state. Depending on the $2b$ input bits it will change its state accordingly. Then, we can pick up codewords diverging from a state of any set P or Q with the aim of trying to minimize the parallel paths and increase the coding gain. Moreover, in order not to increase the complexity in the ML decoder, we need to make sure that all the codewords arriving into a given state are from the same symbols set P or Q.

In Figure 3 and Figure 4, we provide a four-state BPSK and an eight-state QPSK example of our Mo-SOSTTC respectively. We note that the resulting Mo-SOSTTCs

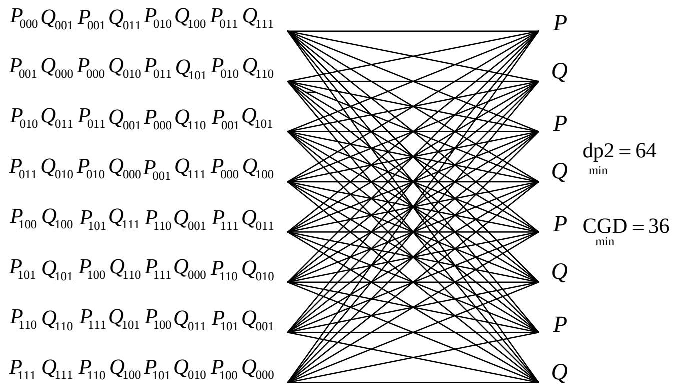

Figura 4. An eight state code for two transmitting antennas; $r = 2$ bits/s/Hz using QPSK.

maintain a minimum CGD as high as SOSTTCs. Moreover, due to the fact that the Mo-SOSTTCs proposed have a grater dp2, this will result in better performance in fast block-fading channels. Using Pseudo-OSTBCs [11], we can systematically design codes for more than two transmitting antennas following a similar procedure.

We propose the design of full-rate Mo-SOSTTC with full diversity and high coding gain for two transmitting antennas

The important properties of Mo-SOSTTC that can be observed are: First, for the diversity gain case, the minimum δ_H (MSHD) of the Mo-SOSTTC is greater that the MSHD of the SOSTTCs. We can see that for both BPSK and QPSK code symbols, in SOSTTCs the $\min \delta_H = 2$ because of using parallel transitions, and the $\min \delta_H = 4$ in Mo-SOSTTC, despite the fact that there are parallel paths on eight-states QPSK-Mo-SOSTTC because each parallel branch has only two codeword matrices with $\min \delta_H = 4$. In addition, we can use a trellis with few states in the Mo-SOSTTCs because we have a full-range difference matrix $\mathbf{D}_k$ between any codeword pair that belongs to $\{P, Q\}$. Secondly, the product distance term dp2 obtained in the Mo-SOSTTCs provides a greater advantage of coding gain as compared to SOSTTCs.

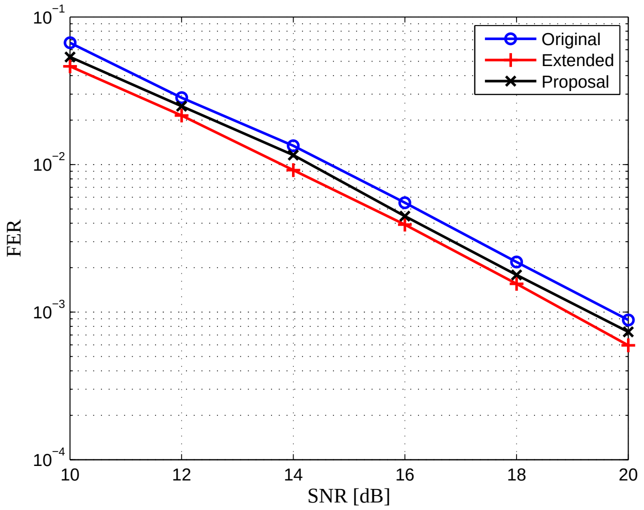

Figura 5. Performance in quasi-static channel for BPSK.

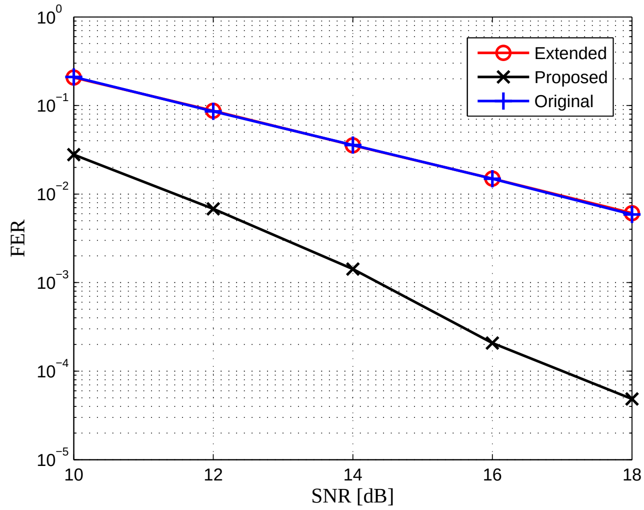

Figura 6. Performance in fast block-fading channel for BPSK.

SIMULATION RESULTS

In this section, we use Monte Carlo simulations to derive the frame error rate (FER) versus the received SNR with two transmitting antennas and one receiving antenna for BPSK and QPSK symbols.

First, we assume a quasi-static flat Rayleigh fading channel, therefore the path gains are independent complex Gaussian random variables with zero mean and variance $\frac{1}{2}$ per dimension and constant during the transmission of one frame. Then, the channel is modeled as fast block-fading, where the channel is constant for each orthogonal space-time block codeword matrix and changes independently from one codeword to the other. In all simulations, a frame consists of 130 transmissions out of each transmitting antenna, and we have $M_t = 2$ transmitting antennas and $M_r = 1$ receiving antenna. For normalization purposes, we consider that $M_t E_s = 1$, thus the noise variance is $\sigma^2 = \frac{1}{2\,\mathrm{SNR}}$.

Figures 5 and 6 show the performance comparison between the four state original SOSTTC [8], the extended-SOSTTC [11] and the proposed Mo-SOSTTC for BPSK (1 bps/Hz) using two transmitting antennas and one receiving antenna for both slow fading and fast block-fading respectively. Figure 5 shows that the performance of our four-state Mo-SOSTTC is between the four-state original and the extended SOSTTC, then the performance for quasi-static fading channel is maintained. Mo-SOSTTC outperforms the original SOSTTC by about 0.45 dB, but it is approximately 0.4 dB worst than that of extended-SOSTTC.

For the fast block-fading channel case, Figure 6 demonstrates that our four-state Mo-SOSTTC for BPSK outperforms both the four-state original and the extended SOSTTC. From the analysis in last section, we can see that the diversity gain of the new Mo-SOSTTC is grater than the diversity of the SOSTTCs, since the minimum Hamming distance is $\delta_H = 4$. Moreover, the product distance dp2 for the Mo-SOSTTC-BPSK is 256, while for both the four-state original and the extended SOSTTC-BPSK dp2 is only 64, then the coding gain is increased.

Since SOSTTC specifically designed for QPSK in the fast fading scenario are not available in the literature then the proposed eight-state Mo-SOSTTC for QPSK is compared with the four-state original and four-state extended SOSTTC, both of which have been designed for quasi-static fading channels. Despite they have different number of states than our Mo-SOSTTC, this is only for comparative purposes. Figure 7 shows the performance comparison between the four-state original SOSTTC [8], the four-state extended SOSTTC [11] and the eight-state Mo-SOSTTC proposed for QPSK (2 bps/Hz) using two transmitting antennas and one receiving antenna in quasi-static fading channel.

For fast block-fading channel, the results are shown in Figure 8. Similarly as the previous BPSK case, the performance is maintained for the quasi-static fading channel case on the Mo-SOSTTC for QPSK, and it is almost identical to both the original and the extended SOSTTC. However, in

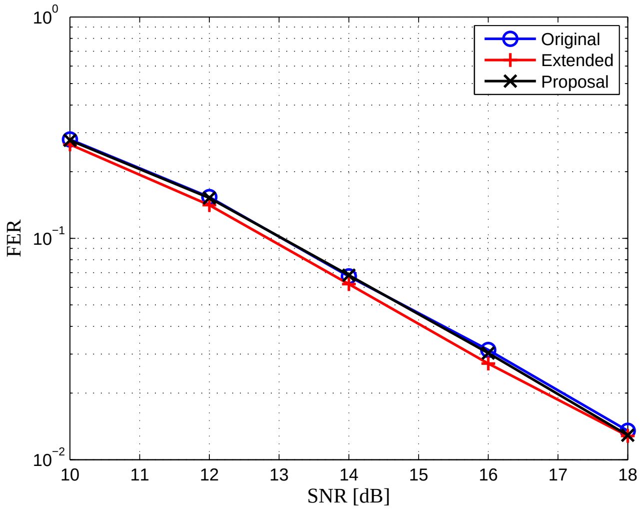

Figura 7. Performance in quasi-static channel using QPSK constellation.

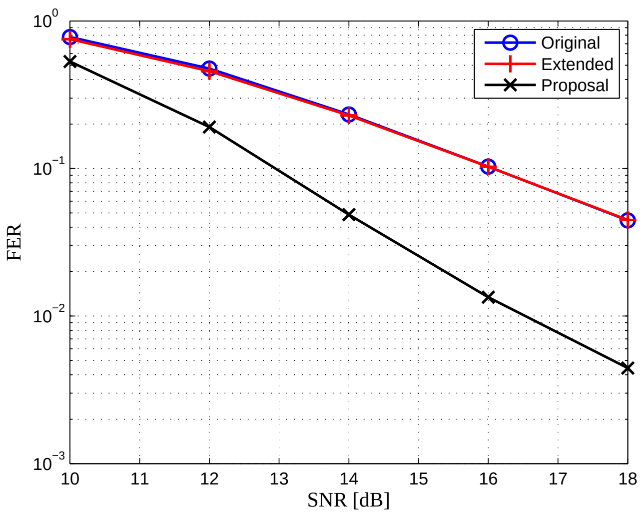

Figura 8. Performance in fast block-fading channel for a QPSK alphabet.

fast block-fading channel our eight-state Mo-SOSTTC outperforms both the original and extended SOSTTC, because the minimum Hamming distance is increased, hence we have a greater diversity. Also, the Mo-SOSTTC presents a dp2 greater than the original and extended SOSTTC, which represents more coding gain than the four-state original and four-state extended-SOSTTC.

CONCLUSION

We have carried out a modification to super-orthogonal space-time trellis codes (SOSTTCs), which is called Mo-SOSTTCs; for both BPSK and QPSK constellations using two transmitting antennas and one receiving antenna for a fast block fading channel, which is modeled as constant for each codeword matrix and changes independently from one codeword to any other. By exploiting the extra rotation parameter at the symbol data in order to obtain a full rank branch difference matrix and decreasing the parallel branches in the trellis, the analysis and simulation results shown that the Mo-SOSTTCs provide full diversity, full rate and higher coding gain as compared to SOSTTCs. Moreover, the complexity of the decoding remains low because of the trellis design structure where for codewords arriving at given state belong to the same orthogonal space-time codewords set. We can obtain higher coding gains using others multilevel constellations like M-QAM.

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Acerca del autor o autores

J. Flores-Troncoso, S. Ibarra-Delgado, R. Sandoval-Aréchiga and J. M. Cervantes Viramontes are with the Faculty of Electrical Engineering, Autonomous University of Zacatecas, Zacatecas, Zac., México 98010
e-mail: jflorest@uaz.edu.mx
salvador6407@hotmail.com
silvergin@yahoo.com
joscervant@yahoo.com
Jaime Sánchez is with the Department of Electronic and Telecommunications, CICESE Research Center, Km. 107 Carretera Tijuana-Ensenada, Ensenada, Baja California México 22860,
e-mail: jasan@cicese.edu.mx
L. Soriano Equigua and J. L. Álvarez Flores are with the Faculty of Mechanical & Electrical Engineering, University of Colima, Colima, Col., México, email: lsoriano,alvarez_jose@ucol.mx